# On Diagnosis and Intervention: Helping Students with Special Needs Learn Fraction Ideas Involving Decimal Numbers

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Despite the emphasis that children should have a robust sense of number and a thorough understanding of fraction (National Mathematics Advisory Panel, 2008), many students continue to struggle with these concepts. Booker Diagnostic Assessment Framework (Booker, 2011) can inform decision about teaching that improves students' learning outcomes. Focusing on decimal numbers, this paper reports how effective use of the framework and instructional materials can help three teachers challenge their students' misconceptions with mathematics, leading to improved learning outcomes.

## Introduction

Since Shulman's (1986) proposed pedagogical content knowledge, learning to teach necessitates efficient use of three micro-skills: general pedagogical, subject matter and pedagogical content knowledge (Borko & Putnam, 1996). General knowledge refers to the broad principles and strategies of general classroom management and instructional strategies needed to ensure a smooth running of lessons. Subject matter knowledge involves key facts, concepts, principles and explanatory frameworks of a discipline, and the type of evidence used to guide the inquiry. It is influenced by the ways in which teachers think about their subject matter and the choices they make in their teaching (Grossman, Wilson, & Shulman, 1989). Pedagogical content knowledge and beliefs are ways of representing and formulating a subject that make it comprehensible to others (Shulman, 1986). It is subject-specific, influenced by individuals' past and present training, teaching and learning experience (Hashweh, 2005). The inexorably intertwined relations between these three aspects of knowledge ensured that teaching is a complex and lengthy process extending for most teachers well after their initial training (Calderhead & Shorrock, 1997).

Compounding the teaching process is an increasing diversity of needs found within classrooms. There is limited information about how these students, many of them having special needs, learn mathematics (Forgasz et al., 2008). This lack of information is due largely to a clear divide between the research interest of mathematics and special education researchers. The former focuses on deep content and pedagogical knowledge, whereas the latter is dominated by a deficit view of disability, relying on experimental design to study basic skills development, memorisation of facts, and drill and practice. From this branch of research comes the most contentious claim, that proficiency in learning among students with special needs is best achieved through direct and explicit instruction. The efficacy of these instructions have been cited and promoted in a range of research papers and policy documents (Apps & Carter, 2006; Purdie & Ellis, 2005). Direct instruction originated in the late 1970s and was deemed inappropriate for fostering the growth of reasoning and higher order cognitive skills subsequently (Palincsar, 1998). The resurgent popularity can be traced to the policy of inclusive education. With a lack of information from mathematics researchers on how best to teach this cohort, it seems that the only credible source of information is found in meta-analysis report such as those produced by Baker et al. (Baker, Gersten, & Lee, 2002) endorsing direct and explicit instruction.

To endorse any teaching approaches one first need to understand what it is one is subscribing to. The fourth edition of *Designing Effective Mathematics Instruction: A Direct*

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*Instruction Approach* (Stein, Kinder, Silbert, & Carnine, 2006) provided a window into the characteristics of such practice. There are 390 basic arithmetic facts: 100 each for addition, subtraction and multiplication of numbers between zero and nine, and 90 division facts for students to learn. In counting, students were taught to use tally marks and the likely difficulty teachers would encounter is when 'students write crooked lines or crowd them together' (*ibid*, p.53). A similar approach was taken to problem solving, where the focus was on recognising key words and word patterns instead of developing the thinking needed for problems solving. Such practice not only undermines mathematics as a discipline, it also neglects the importance of conceptual understanding and fails to acknowledge students' rights and ability to construct mathematics of their own.

Criticisms about the sole reliance on well-controlled experimental design have come from within the field of special education. However, much of the arguments remain mere rhetoric without any constructive suggestion for improvement. These conflicts in the philosophical and conceptual frameworks have produced an epistemological and ontological stalemate (Gallagher, 2007), or what Kauffman et al. (Kauffman, Nelson, Simpson, & Mock, 2011) termed as 'permanent fixtures' of special education. Such a stalemate can no longer provide new insight into how students with special needs learn. Instead, a framework that builds on a well-established mathematical epistemology that addresses students' mathematical misconceptions is needed. Meanwhile, Norwich and Lewis (2007) argue that students with special needs do not need a qualitatively different pedagogy. Perhaps good teaching that begins with accurate diagnosis of students' misconceptions, combined with an integration of subject matter knowledge, pedagogical content knowledge, can improve the learning outcomes of these students.

## Theoretical Framework

This study posits that knowledge is actively created in social situations (Booker, Bond, Sparrow, & Swan, 2010) and distributed across people, environment and objects (Pea, 1993). Mathematics relies on a system of symbolic representations to express its meaning. These representations, in the form of digits, signs and objects, are not always 'transparent' to the learners (Cobb, Yackel, & Wood, 1992) and therefore could not be mastered merely through drill and practice. Instead, any mathematical concept begins with providing full meaning of number words to match instructional representations, and later linking it with the symbols that represent them (Booker et al., 2010). This makes the design of effective instructional representations and learning sequence to bring out the abstract nature of mathematics, and ways to construct understanding vital. Based on Booker Diagnostic Assessment Framework (Booker, 2011), effective diagnosis of students' difficulties with mathematics begins with designing tasks that relate to mathematical conceptual frameworks and not mere procedural accuracy. Such tasks provide insights into students' underlying thinking pattern, including what is known, what needs to be known and gaps in a student's thinking. Once students' misconceptions are established, appropriate teaching instructions can be developed to mediate learning success. Specifically, instructions on decimal numbers began with an acknowledgement that students' prior knowledge about whole number would have substantial impacts on their learning about decimal numbers and this is unavoidable (Ni & Zhou, 2005). Instead, the aim of instructional design was to create various opportunities to explicitly confront the conceptual conflicts students' prior knowledge about whole number has with fraction numbers. These conflicts include knowing that (a) decimal numbers are a composition of whole numbers and fractions broken by the decimal point; (b) its language ends with 'th' and reading it differs from reading whole numbers as there is no constant unit that correspond to ones; (c) the 10-1 relationship in decimal numbers begin with tenths and decrease in quantity towards the right, whereas the same relationship in whole numbers increase in quantity towards the left; (d) a zero is a placeholder and when it is at the extreme right on a decimal number, its value unchanged.

## Method

This study was conducted in a high school located in a low socio-economic area in Brisbane, Australia. The participants were three teachers and their Year 8 and 9 students with special needs. Both Ted and Hana are experienced teachers in special needs and English respectively whereas Ann is a beginning teacher with no mathematics training. 12 Year 9 students had been identified by a medical practitioner as having a developmental disability under the Education Queensland guidelines and attended a separate program for Mathematics. 19 students were identified through the school's internal screening process as having a learning difficulty and attended a support program that lasted one term. Using design research method, the study began with diagnostic assessments of students' decimal numbers knowledge. The data collected were then used to design instructional sequence based on Booker et al.'s (2010) connected, conceptual understanding of learning mathematics. The aim was to help teachers teach decimal number place value, renaming and sequencing skills. In particular, instructional games were used to (a) provide a platform where the concept of decimal numbers could be constructed in a non-threatening matter, (b) develop teachers' subject matter and pedagogical knowledge, and (c) create a rich learning environment. Ongoing daily analysis of teachers' lesson presentations and student's participation in classroom activities further shaped and refined subsequent lessons.

A total of 50 lessons, spanning across two terms for the Year 9 class and 5 weeks for the Year 8 classes, were planned, observed, and redesigned. Year 9 class taught by Ted received 24 lessons, Ann of Year 8A 15 lessons and Hana of Year 8B 11 lessons. Each teacher was briefed on the mathematical basis of each learning activity and ways of using the instructional materials. Individually, the teachers decide how to introduce each concept, what examples to provide in order to help students comprehend the concept, and how to present and facilitate the activities. The data presented here include students' performance on *Booker Decimal Fraction Test* A and B (Booker, 2011), video recording of the lessons, and teachers' feedback. The tests were administered to all three classes at the beginning and the end of the project. The 10 questions assess students' understanding of basic concept, naming tenths and hundredths, sequencing, ordering, computing and solving word problem. The initial data collected contribute to the lesson design while the results obtained from Test B provide an indication of lesson effectiveness.

## Results and Discussion

The graphs presented in Figure 1 provided comparison results for each class for Test A and B. The initial data revealed that most students had difficulty naming decimal numbers involving hundredths. They also could not sequence, order, compute and solve word problems and many similar errors were made in the test. For example, while most students had no difficulty writing 3 and 7 tenths as 3.7 (question 2a), some had trouble understanding 8 tenths as 0.8 (question 2b) and wrote it as 8.0, 8.10 or 80 instead. A majority of students were unable to name decimal fractions involving hundredths especially when an internal zero was involved. Thus, 7 and 2 hundredths was written as 7.2 or 7.200 (question 3a) and 16 hundredths as 1.6 or 16.0 (question 3b).

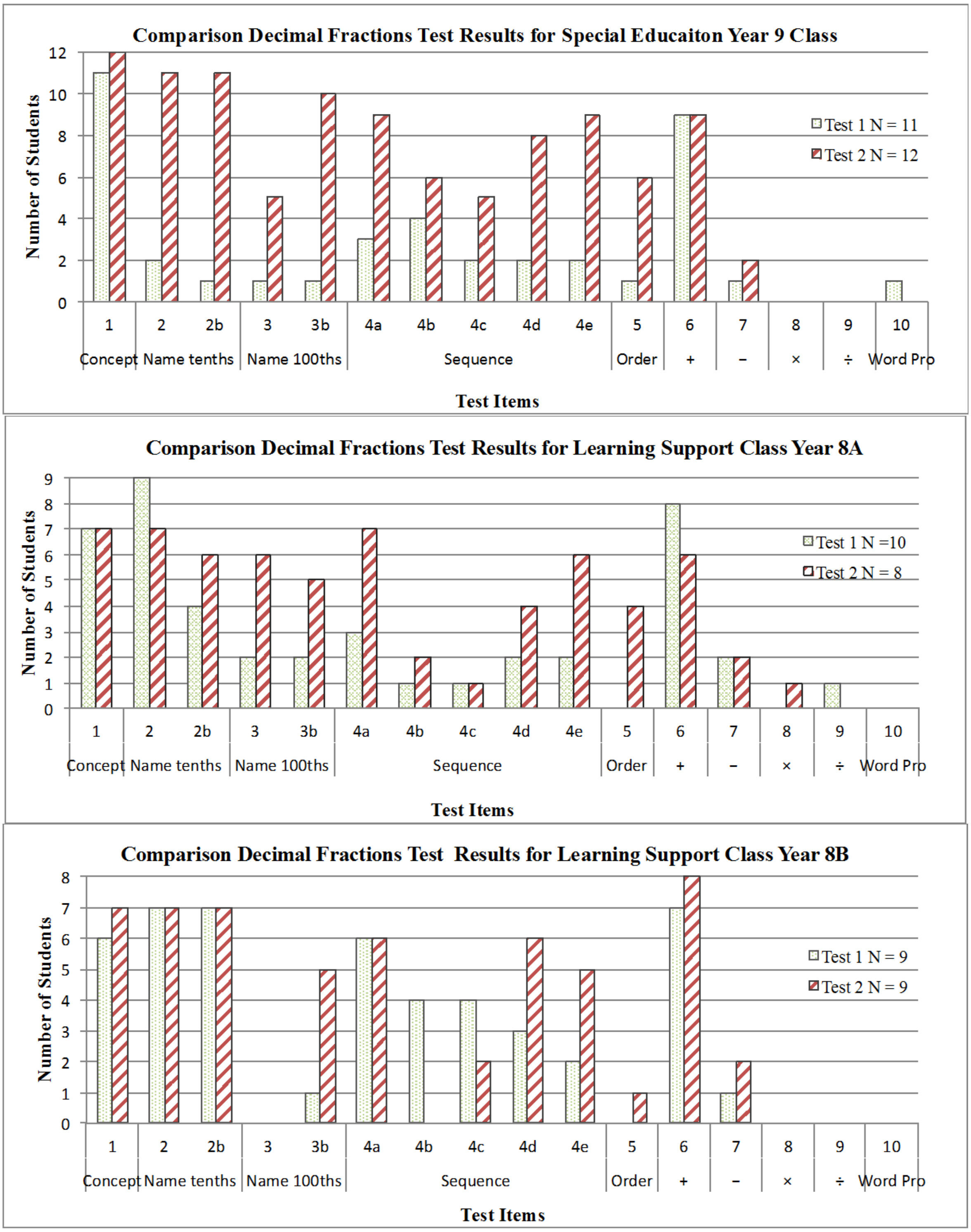


*Figure 1.* Comparison results of decimal fraction test across three classes.

This sort of error, caused mainly by a lack of decimal fractions place value understanding, was particularly evident when attempting to sequence decimal fractions. When asked to write the numbers that are 3 tenths more from the corresponding set of decimal fractions (question 4), many students wrote: (4a) 8.4 as 11.4, (4b) 9.7 as 9.12, (4c) 6 as 9, (4d) 8 as 0.11, and (4e) 2.9 as 5.9. Ordering decimal fractions from least to greatest (question 5) was the most challenging task apart from computation. A majority of students wrote 3.0, 3.2, 3.04, 3.10, and 3.16 or the reverse, 3.16, 3.10, 3.4, 3.2 and 3.0. It would appear that these students assumed ‘16’ is bigger than 10, 4 and 2 and responded accordingly. A lack of place value understanding also hindered students’ ability to compute with and solve problems involving decimal numbers. The varying teaching durations among the three classes reflect the nature of design research and in part have contributed to large gains made between tests in focus areas. Further, six out of nine students in Year 8B were recent arrivals holding refugee status. The need to adjust to a different form of schooling may have influenced their learning.

Although the results identified students’ mathematical difficulties, the ‘why’ and ‘how’ questions were not obvious to participating teachers. As such, the instructional design served both to provide the subject matter and the pedagogical content knowledge needed to teach decimal numbers. The subsequent classroom research further enabled the teacher and researcher to comprehend learning from the students’ point of view. Drawing on the initial results, the instructional sequence focused on decimal number place value, renaming and sequencing skills. These findings are addressed below.

### *The Concept of Place Value and Renaming*

The introduction of decimal numbers began with finding numbers smaller than one, as represented by the base 10 blocks. Students in all three classes could only thought of ‘*halving it*’ or cut it into quarters. Only when the teacher reminded students the 10-1 relationship learned in whole numbers could students comprehend the logical process of breaking 1 one is 10. Once this was established, the ones and tenths and ones and hundredths grids (Figure 2) were used as instructional representations to develop the language and associated symbols. When showed 1 tenth on the tenths grid, many students straightaway said, ‘*it’s one out of ten*’ but unable to write it as symbols. Teachers in all three classes had to use the ones block with the grids and a place value chart to show the writing for 1 tenth is 0.1.

*Figure 2.* Using the grids and place value chart to show the difference between 1 tenth and 1 hundredth.

The introduction of 1 hundredth followed similar instructional path of breaking each tenth into ten parts, resulting 100 parts. When asked how to write it as a number, Ann’s student Mondi insisted that it should be written as 0.100, with ‘*two zeros*’ added at the back. When asked what is the meaning of zero? The range of responses included ‘*it’s a number*’, ‘*it’s zero*’ ‘*it’s nothing*’, ‘*you can times it*’ or ‘*you can add on to other number like twenty*’. The concept that zero is a placeholder was discussed when students agreed it was erroneous to write 975021 as 97521. Once that was established, students were then ready to accept that 1 hundredth should be written as 0.01 and not 0.100.

Making sense of numbers is a critical aspect of learning. To have number sense, students need to have full understanding of number concepts such as zero. The other skill is to be able to rename numbers in equivalent forms (Booker et al., 2010). In fractions, it does not restrict to naming 1 third as 2 sixths, 3 ninths, but also knowing that 2 and 3 tenths can be read as 23 tenths and written as $2\frac{3}{10}$ or 2.3. Bingo games were designed where the printed words 2 and 6 tenths (or 26 tenths) can be matched with either the symbol 2.6 or the ones and tenths grid. In this way, decimal numbers are no longer symbols devoid of meaning but words that express their values and can be ‘seen’ on the grids. Numbers that involves hundredths provided further opportunities to challenge whole numbers conceptual conflicts and extend students' understanding of place value. Whereas students could guess the answer for game cards that are restricted to ones and tenths, such as 224 hundredths as 2.24, cards that involved hundredths required deeper knowledge of the number system since 30 hundredths does not look quite the same as 0.3.

Given that renaming was new to both the teachers and students, the instructional games provided both the subject matter and pedagogical knowledge to help teachers and students construct the understanding. Expression such as '*Oh gosh, zero point seven is 7 tenths!*' could be heard when students came to the 'discovery' on their own through the games. Teachers too, benefited from the games, as Ann remarked, '*All my life I'm afraid of mathematics. I was learning the game as they were learning the games. I feel much more equipped to lead them on this learning journey and I've a better understanding now.*'

### *Comparing, Sequencing and Ordering Decimal Numbers*

Teaching students to sequence numbers quickly brought to the realisation that they could not count decimal numbers. Many students simply spelled out the words, 'zero point seven, *zero point eight, zero point nine*', which naturally led them to say '*zero point ten*' and wrote 0.8, 0.9, 0.10. In one activity, students have been working on adding 3 tenths more to 2.3, 5.7 and 3.9. These the students completed with ease. However, when asked to add 3 tenths more to the number 4, everyone replied seven. Instead of seeing the numbers as a set, their focus was entirely on the last digit. This realisation helped teachers to change their instructions, reading mathematics as a language such as 2 and five tenths instead of spelling it out so students could determine the number magnitude. In one activity, Ted noticed that his student Sam had great difficulty finding the difference between 6.9 and 7.7. Only when Ted read it as 77 tenths and 69 tenths did Sam able to comprehend the task.

### *Teacher General, Subject Matter and Pedagogical Content Knowledge*

The relationships between general, subject matter and pedagogical content knowledge are intimately linked in the diagnosis and teaching process. Teachers' subject matter knowledge greatly influenced how they perceived students' response and their subsequent instructional choice. For example, Hana of Year 8B asked her students, '*What is a fraction?*' to which Jama replied, '*There's a numerator and a denominator.*' Hana was pleased with the response and did not pursue the issue further. Subsequent lessons and continuing probing quickly revealed that what Jama knew was 'naming' the fraction parts, and not a conceptual understanding of the part/whole relationship. Likewise, when students repeatedly wrote 5.9 and $\frac{5}{9}$ or 1.2 and $\frac{1}{2}$ as examples for fractions, there is a need to probe further to determine if the use of similar digits was coincidental or a misconception of treating the decimal point as the vinculum.

Using instructional representations in the teaching process was not straightforward, as the teachers tended to briefly mention them and focus extensively on using symbols to explain. Cobb et al. (1992) argues that many teachers assume that students will automatically construct the correct internal representations from the materials presented and understood 'the mathematical relationships they are to construct before they have constructed them' (p. 5). What teachers failed to realise is that materials that are thought to be 'transparent' – where their meanings are obvious, is the product of teachers' own understanding. For many students, the meanings are not obvious. In one activity, students were asked to shade decimal numbers next to the corresponding grids. Many students across all three classes shaded 2 and 4 hundredths for 2.4 on the task sheet. Ted first tried to explain that the tenths is bigger and the hundredths is smaller because of their position on the place value to no avail. It was only when both numbers were represented on the grids that the difference between them was made clear. This experience made Ted realise, '*It wasn't just about teaching a concept from the board, but also how to transfer what's taught to the physical, mental, knowledge of the terminology*'.

The use of questioning technique to elicit students' underlying thinking is another important diagnostic and teaching skill. For Hana, the classroom research helped develop a task-analytic approach to teaching by breaking learning down to teachable steps. She admitted that initially, she thought, '*the program is basic, is easy and the lesson is going to drag,*' but soon appreciated that each sub-construct needed to be carefully introduced and cemented before introducing a new construct. She learned to (a) check for understanding at every step of the lesson, (b) ask many questions at the right level to ensure a high success rate, and (c) build confidence and monitor closely the work being done by each student. Ann too, learned to probe for further understanding. She remarked,

> 'You ask, "What do you mean by that? Oh ok… so you think it could be…" Whatever the answer is and you know it is wrong but don't say that's wrong. You said, "Oh ok, and how did you get to that? Ok… could you show me how you did that, what processes did you use?" And then they correct themselves… and they beam'.

Prior to the study, all three teachers mentioned managing students' non-compliance as their most challenging task when teaching this cohort. Students in all three classes were quite restless during the initial study. Some questioned why they had to be removed from their regular class. The use of instructional games and representations to help students learn the concept soon eliminated many behavioral problems. Ann remarked,

> 'No behaviours, the same kids who had been suspended and on the red book. They're committed to their learning. They want to learn.' Hana agreed, 'They're experiencing success and they want to come. They didn't want to go back to their regular class. The lessons just go… I wish it's the same with literacy!'

## Conclusion

Students' misconceptions with decimal numbers reported here are not new and have been well documented in the research literature. The value of this data lies in the fact that these students are ascertained to have either a learning difficulty or disability, such as intellectual impairment, and Autistic Spectrum Disorder. The result points to the nature of mathematics and the way it was instructed as a cause of students' difficulty, and not as a result of student deficit. In particular, good teaching that challenges students' misconceptions and guides them to construct their own mathematics, as well as the time allocated to learn a particular topic is of the essence. Booker (2011) maintains that well-focused assessment could reveal students' underlying belief systems and provide guidance to improve teaching. In this case, the results obtained enabled research to design specific activities that highlighted misconceptions and built understanding. More research is clearly needed to extend our understanding of teachers' subject matter and pedagogical content knowledge and students' mathematical thinking at the classroom level to effect change.

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